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Specific Impulse Calculations for Air-Breathing Propulsion

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Nomenclature

A	=	area
AA	=	air augmentation ratio
F	=	thrust
g	=	gravitational acceleration
Is	=	specific impulse
O/F	=	ratio of propellant oxidizer to fuel
P	=	pressure
V	=	velocity
$\dot{w}$	=	weight flow rate

Subscripts

e	=	exit
f	=	fuel
j	=	jet
m	=	mixture
n	=	net
o	=	oxidizer
p	=	propellant
$p \cdot n$	=	propellant net
∞	=	freestream conditions

Introduction

THE rapid rise in the number of potential applications of air-breathing rocket propulsion systems to low-altitude hypersonic missions has caused the writers to develop a quick and simple method for calculating performance of propellants for these systems. This note describes a method for making air-breathing propellant performance calculations with the assistance of chemical equilibrium composition computer programs of the type normally used for rocket propellant performance calculations.

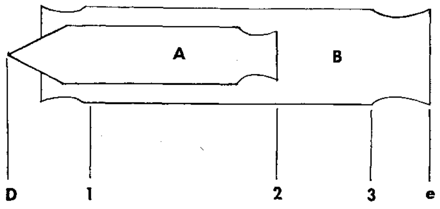

Fig. 1 Combustion apparatus model.

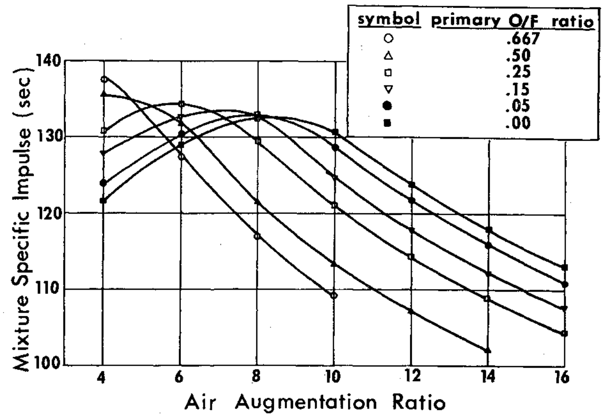

Fig. 2 Intermediate I_s vs air augmentation ratio.

The theoretical model for this calculation method is shown in Fig. 1. The onboard propellants are reacted in a primary combustion chamber (A), and the resultant fuel-rich products are then exhausted into the main combustion chamber (B). In the main combustion chamber (B), the fuel-rich products are subsonically reacted with ram air and are then expanded through a supersonic nozzle to the atmosphere. The working substance in the main combustion chamber is considered to be homogeneous and invariant in composition as it passes station 3 (Fig. 1). The combustion pressure in the main chamber never exceeds the ram recovery pressure. And, for the examples used in this note, the main chamber combustion pressure was defined as being exactly equal to the maximum ram recovery pressure.

The net specific impulse for the onboard propellant under given conditions of air augmentation is readily derived from the output of the previously mentioned computer program.

Method

The chemical equilibrium composition program used for this method requires chemical descriptions of the propellant ingredients including air, a statement of the enthalpy of each of the propellant ingredients, and the relative amounts of fuels and oxidizers used. The ram air is treated as a portion of the oxidizer fraction. Air elements, such as nitrogen and oxygen, normally have zero for heats of formation, but in ramrocket and ramjet systems the rocket has done work on

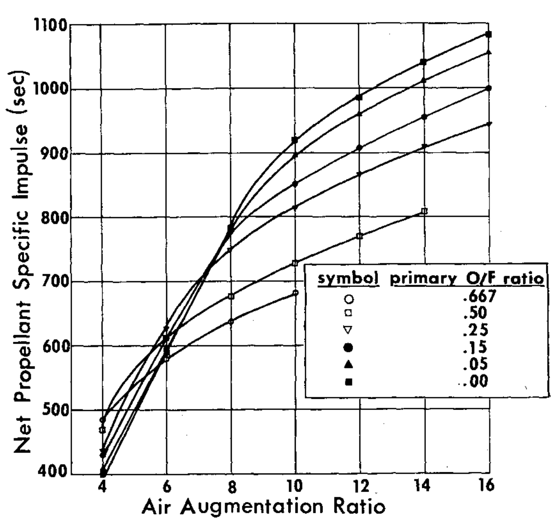

Fig. 3 Propellant I_s vs air augmentation ratio.

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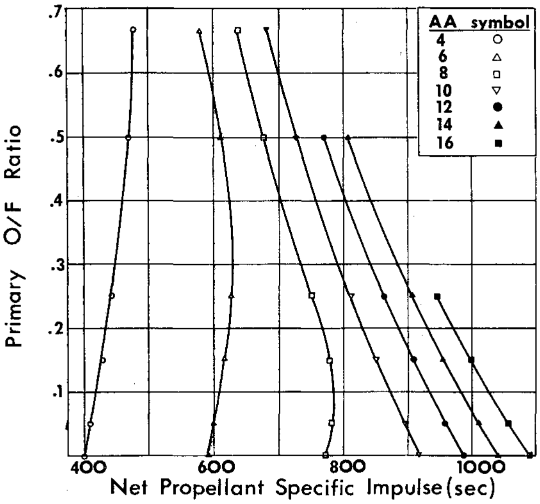

Fig. 4 Propellant I_s vs propellant composition.

the ram air, and this energy is added to the nitrogen and oxygen as heats of formation data. The specific impulse values, output by the computer program are then based on the mass flux of the total mixture, fuel, oxidizer, and air. The computer program output is then converted to net specific impulse based on the flow rate of the airborne propellant.

The specific impulse output by the computer program ($I_{s(M)}$) may also be called the mixture specific impulse output and is defined as

$$(I_{s(M)} = F_j/(\dot{w}_o + \dot{w}_f + \dot{w}_{\text{air}})$$

In this note, air augmentation ratio is defined as

$$AA = \dot{w}_{\text{air}}/(\dot{w}_o + \dot{w}_f)$$

Therefore, the specific impulse based on onboard propellant flow rate is

$$I_{s(p)} = (1 + AA)I_{s(M)}$$

A consideration of system energy shows that the net thrust of an air-augmented system may be equated as

$$F_n = (P_eA_e + \dot{w}_mV_e/g) - (P_eA_\infty + \dot{w}_{\text{air}}V_\infty/g) - P_e(A_e - A_\infty)$$

When the ramrocket's exhaust stream is optimally expanded to ambient pressure,

$$F_n = (\dot{w}_mV_e/g) - (\dot{w}_{\text{air}}V_\infty/g)$$

However,

$$F_j = \dot{w}_m V_e/g$$

$$I_{s(P,N)} = (1 + AA)I_{s(M)} - AA(V_\infty/g)$$

Several cases were calculated using RFNA/MAF-4 as the onboard propellant system. In the calculated cases the flight condition was Mach 1.5 at sea level. Figure 2 is a plot of mixture specific impulse values vs air augmentation ratios for several onboard propellant O/F ratios. Figure 3 is a presentation of net propellant specific impulse vs air augmentation ratios for the several onboard propellant O/F ratios. Figure 4 presents the same information as Fig. 3 but with a different emphasis.

Discussion

Presently available chemical equilibrium compositions computer programs can be used for the calculation of ramrocket and ramjet propellant performance data. The chemical equilibrium composition programs usually concern themselves only with the combustion products flow as it enters and passes through a supersonic nozzle. The method developed in this note permits calculation of maximum net specific impulse values through treatment of the ram air as a portion of the oxidizer system and by considering that the combustion products are homogeneous mixtures during ideal supersonic expansions.

Figure 2 notes that the mixture specific impulse curves understandably shift toward higher air augmentation ratios as the airborne propellant system becomes increasingly fuel-rich.

Figure 3 presents net specific impulse vs air augmentation ratios, and here the performance curves demonstrate crossovers as air augmentation increases. At low air augmentation ratios the onboard propellant streams with the highest O/F ratios possess the larger specific impulses. As the air augmentation ratios are increased, those onboard propellant streams with the smallest O/F ratios have greater thrust capability.

Figure 4 highlights the influence of the onboard propellant's O/F ratio on net specific impulse for given air augmentation ratios. This mode of presentation is useful in making air augmentation tradeoff studies.

The method of calculation described in this paper applies equally well to ramjet propellant performance determinations. In a ramjet system, the onboard propellant consists only of fuel, and the onboard propellant O/F ratio for these cases is zero, resulting in a single curve for each fuel considered.

The examples used here featured a liquid onboard propellant system of red fuming nitric acid and a mixed amine fuel. Solid propellants can also be used as onboard propellants with the same chemical equilibrium compositions computer program.

In conclusion, the method described here greatly extends the utility of presently available chemical equilibrium compositions computer programs. These programs can now be employed to make ramrocket and ramjet propellant performance calculations in addition to their original rocket propellant performance uses.

Buckling of Circular Conical Shells under Uniform Axial Compression

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Nomenclature

a	= distance of the top of a truncated cone from the vertex, along a generator
C_n	= displacement coefficient of the nth term of w
E	= modulus of elasticity
h	= thickness of shell
H_2, H_2^{-1}	= differential operators defined by Eqs. (6) and (48) of Ref. 3
K^4	= $12(1 - \nu^2)(a/h)^2$
m, n	= integers
P	= axial force
P_{cyl}	= axisymmetric buckling load of corresponding cylindrical shell
u, v, w	= nondimensional displacements, obtained by dividing the physical displacements by a
x	= nondimensional axial coordinate along a generator

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